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The stability of steady convective motion in a vertical slender slot with non-uniform volumetric energy sources and unequal surface temperatures

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Abstract—The problem of stability of a liquid layer contained between two vertical plates at different prescribed temperatures and subjected to an external radiation source penetrating its body is solved. Results for the effects of radiation and the boundary temperature difference are presented in graphical form in terms of two different critical Grashof numbers as functions of the Prandtl number, the optical thickness of the fluid and the wall reflectivities. A physical justification for the behavior of instability under wide range of each parameter is given. Conclusively, the effects of radiation parameters on the stability of liquids are found to be contrary with that reported on radiating gases under the same isothermal case.

INTRODUCTION

THERMAL convection in a fluid layer between two parallel plates with internal heat sources is of great interest in numerous applications such as absorbing liquid solar collectors and nuclear reactor design and safety problems. Such systems involve direct absorption of the external radiation by the working fluid. The resulting heat source functions, arising from the distribution of the absorbed radiation within the fluid, are essentially exponential and decrease monotonically from the transparent to the opaque surfaces. These functions may range from a uniform distribution to an impulse function at the transparent (radiating) boundary, depending on the optical thickness of the layer. A review of the literature on the stability of the fluid due to internal heat generation indicates that for the case of a horizontal layer, the problem is examined both experimentally [1–5] and theoretically under different hydrodynamic and thermal boundary conditions [6–11]. In the case of the vertical orientation of the absorbing layer, the published work is restricted to convection generated by uniformly distributed heat generation as reported in references [12–14]. In this case, the steady flow is characterized by even velocity and temperature profiles, unlike the flow between the heated planes at different temperatures which is prescribed by odd profiles [15]. These results show strong thermal effects on the stability such that as Pr increases the critical Grashof number GI_c is greatly decreased, reaching an asymptotic value of about $3.12 \times 10^4 \, Pr^{-1/2}$ as $Pr \to \infty$. The corresponding limiting value of GE with the heated planes being at different temperatures is defined as $GE = 9.4 \times 10^3 \, Pr^{-1/2}$ [16]. The stability of radiating gases contained inside slender slots and subjected to either prescribed wall temperatures or convective boundary conditions have also been studied [17–19].

The aim of the present work is to study the stability of a vertical layer of fluid due to non-uniform internal heat sources and unequal surface temperatures.

ANALYSIS

The basic model consists of a fluid layer of thickness h, contained in a vertical narrow slot. A Cartesian coordinate system (x, y) is chosen such that the x axis is vertical with the origin in the middle of the slot. Constant temperatures T_1 and T_2 and surface reflectivities r_1 and r_2 are prescribed at the surfaces $y = 0$ and $y = h$, respectively. The surface at $y = 0$ is subjected to a normal incident shortwave radiation of intensity I_s, attenuated in fluid as penetrating in its body. In this problem, with working fluids considered as liquids which are opaque to infrared radiation, the emission is negligible. In the absence of suspended particulates, scattering effect can also be neglected, or lumped with the absorption of the fluid into a single attenuation coefficient as considered here in this analysis.

For the non-dimensional description of the problem the physical quantities of length, velocity, temperature and time are measured in units of d, $\gamma g \Delta T d^2/\nu$, ΔT and d^2/ν, respectively. Accordingly, the Navier–Stokes equations for the velocity vector V and the heat equation for the temperature θ, with the Boussinesq approximation, are given as

$$\nabla \cdot V = 0 \tag{1}$$

$$\frac{\partial V}{\partial t} + GE(V \cdot \nabla V) = -\nabla p + \nabla^2 V + \theta \mathbf{j} \tag{2}$$

$$\frac{\partial \theta}{\partial t} + GE(V \cdot \nabla \theta) = \frac{1}{Pr} \nabla^2 \theta + \frac{1}{Pr} Q_v \tag{3}$$

where, $\mathbf{j}$ = unit vector in vertical (x) direction, p = dynamic pressure, Q_v is the dimensionless volumetric heat source, resulting from the attenuation by absorption of the penetrating radiation which can be obtained from the solution of the radiative equation as [11]

$$Q_v = -\tau \frac{GI}{GE} \left(\frac{e^{\tau Y^*} + r_2 \, e^{-\tau Y^*}}{e^{\tau} - r_1 r_2 \, e^{-\tau}} \right); \quad Y^* = 1 - Y. \tag{4}$$

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NOMENCLATURE

a	wavenumber
C_i	c_i/U_0, non-dimensional wave speed
c_i	wave speed
D	d/dY, operator
g	gravity
GE	$=\gamma g(T_2-T_1)h^3/\nu^2$, external Grashof number
GI	$=\gamma g I_s h^4/k\nu^2$, internal Grashof number
h	thickness of fluid layer
I_s	normal external radiation leaving the transparent surface
k	thermal conductivity
p	pressure
Pr	Prandtl number
Q_v	dimensionless volumetric heat source
T_1, T_2	temperatures of the transparent and opaque surfaces respectively
t	time
$\bar{U}$	velocity component in the vertical direction
U_0	$=g\gamma I_s h^3/k\nu$, characteristic thermal velocity
V	velocity vector
X, Y	$=(x, y)/h$, non-dimensional coordinates
x, y	coordinates with y measured normal to the fluid layer and x parallel to it.

Greek symbols

β	volumetric coefficient of absorption of the fluid
γ	coefficient of thermal expansion of the fluid
θ	dimensionless temperature
ν	kinematic viscosity
τ	$h\beta$, optical thickness
ψ	dimensionless perturbed stream function.

Subscripts

m	maximum quantities.

Superscripts

$\bar{\ }$, $'$	mean and perturbed quantities.

Equations (1)–(4) show that the physical properties of the system are described by six non-dimensional parameters, namely the external Grashof number GE, the internal Grashof number GI, the Prandtl number Pr, the optical thickness τ and the reflectivities, r_1 and r_2.

The variables V and θ satisfy the following boundary conditions for rigid boundaries (i.e. no-slip)

$$V = \theta = 0 \quad \text{at} \quad Y = 0 \tag{5}$$

$$V = \theta - 1 = 0 \quad \text{at} \quad Y = 1. \tag{6}$$

The steady state basic solution of equations (1)–(4) consists of a temperature distribution governed by conduction only and a plane parallel flow in the vertical direction. Using conditions (5) and (6), the basic solution can be found as

$$\bar{U} = U_1 + \frac{GI}{GE} U_2 \tag{7}$$

$$\bar{\theta} = \theta_1 + \frac{GI}{GE} \theta_2 \tag{8}$$

where

$$U_1 = \frac{-1}{12} Y(Y-1)(2Y-1)$$

$$U_2 = C_1 U_1 + C_2(Y^2 - Y) + \frac{1}{\tau^2 \cdot F}[(e^{\tau Y*} + r_2 e^{-\tau Y*}) - C_3 Y - C_4]$$

$$\theta_1 = Y$$

$$\theta_2 = \frac{1}{F\tau}[C_4 + C_3 Y - (e^{\tau Y*} + r_2 e^{-\tau Y*})]$$

where

$$C_1 = [(1+r_2) - (e^{\tau} + r_2 e^{-\tau})]/F$$

$$C_2 = 6[(e^{\tau} - 1) + r_2(1 - e^{-\tau}) - C_3\tau/2 - C_4\tau]/(\tau^3 \cdot F)$$

$$C_3 = C_1 \cdot F, \quad C_4 = (e^{\tau} + r_2 e^{-\tau})$$

$$F = (e^{\tau} - r_1 r_2 e^{-\tau}).$$

To describe the secondary solution corresponding to the convective state of the system, the governing equations are perturbed through two-dimensional infinitesimal disturbances of the form

$$F(X, Y, t) = \bar{F} + F'(Y)\, e^{ia(X+ct)} \tag{9}$$

where $F = V$, p or θ denote the perturbed flow quantities, $\bar{F}$ and F' refer to base flow and amplitude of perturbed flow quantities, respectively. a is the real wavenumber in the X-direction, and c is the complex growth rate.

Introducing (9) into equations (2) and (3), eliminating the pressure by cross-differentiation, then defining a stream function ψ', that satisfies (1), we finally obtain

$$D_1^2(D_1^2 - iac)\psi' - iaGE(U_1 D_1^2 - D^2 U_1)\psi' - iaGI(U_2 D_1^2 - D^2 U_2)\psi' + D\theta' = 0 \tag{10}$$

$$\left(\frac{1}{P_r} D_1^2 - iac\right)\theta' - iaGE(U_1\theta' - D\theta_1\psi') - iaGI(U_2\theta' - D\theta_2\psi') = 0. \tag{11}$$

Here the operator notations are defined as

$$D = \frac{d}{dY}, \quad D^2 = \frac{d^2}{dY^2} \quad \text{and} \quad D_1^2 = D^2 - a^2.$$

The variables ψ' and θ' satisfy the homogeneous boundary conditions

$$\psi' = D\psi' = \theta' = 0 \quad \text{at} \quad Y = 0, 1. \tag{12}$$

The eigenvalue problem (10)–(12) must be solved numerically in order to determine the spectrum of the characteristic disturbances and the boundary of the flow stability. Galerkin's method proves adequate for this purpose. Only a brief outline of the solution method will be given here. For details see reference [20]. The disturbance amplitudes of the functions ψ' and θ' are expanded in N-term expansions in the form of orthogonal eigen functions, satisfying their boundary conditions (12). Accordingly

$$\psi' = a_1 q_1 + a_2 q_2 + \cdots + a_N q_N \tag{13}$$

$$\theta' = b_1 p_1 + b_2 p_2 + \cdots + b_N p_N \tag{14}$$

where q_n and p_n are taken as

$$q_n = \frac{\cosh aY - \cos e_n Y}{\cosh a - \cos e_n} - \frac{\sinh aY - a \sin e_n Y/e_n}{\sinh a - a \sin e_n/e_n} \tag{15}$$

$$p_n = \sin(n\pi Y), \quad n = 1, 2, .., N \tag{16}$$

and e_ns are the positive roots of

$$(\cosh a \cos e - 1) + \frac{e^2 - a^2}{2ea} \sinh a \sin e = 0. \tag{17}$$

When the approximate solutions for ψ' and θ' are introduced into the set of the differential equations (10) and (11) and the orthogonality conditions are utilized, a set of linear algebraic equations is obtained in the matrix form

$$(CI - A)\mathbf{X} = 0. \tag{18}$$

Here, $\mathbf{X} = (a_n, b_n)^{\mathrm{T}}$ is the transpose of the coefficients vector associated with N-term expansion, I is a unit matrix, A is a matrix of $2N \times 2N$ resulting from orthogonalization, and C is the eigenvalues of the system which are functions of the parameters GE, GI, Pr, τ, r_1, r_2 and the wavenumber a. Equation (18) is solved numerically using the complex Q–R algorithm to determine the characteristic eigenvalues C in terms of the given system parameters. The accuracy of the numerical solution obtained by the Galerkin method was checked for a different number of expansions N. The comparison shows that the use of N from 8 to 14 is very sufficient to get an excellent convergence criteria less than 0.5% over the range of the parameters considered.

RESULTS AND DISCUSSIONS

Consideration is first given to the base flow curves for selected values of the optical thickness of the fluid, τ, under the condition of $GE = 0$ (Figs 1 and 2). Primary interest in these figures is in the influence of τ on the position and magnitude of the maximum fluid temperature, $\bar{\theta}_m$ (minimum density) which characterizes the behaviour of the induced flow inside the slot.

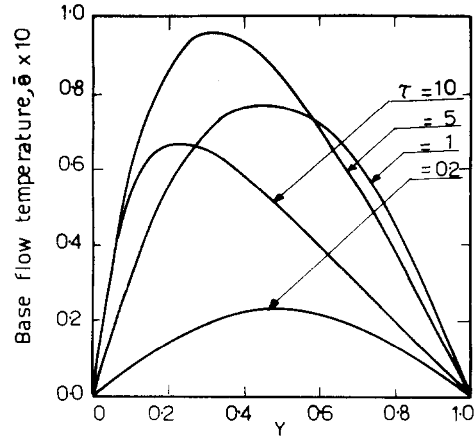


FIG. 1. $\bar{\theta}$ vs layer thickness Y for different values of τ($GE = 0$, $r_2 = 0$).

For $\tau = 0.2$, the heat source distribution is almost uniform and the corresponding steady flow is characterized by even temperature and velocity profiles. As τ is increased, say from 0.2 to 5, the position of $\bar{\theta}_m$ is moved far from the midplane, owing to this antisymmetric heating of the fluid. This causes a transition in the flow regime from two-parallel cells to a single cell. The temperature level $\bar{\theta}_m$, which is proportional to the velocity level, is observed at first to increase and then to decrease with τ. The optical thickness is then expected to have a destabilizing–stabilizing effect on the flow field over its range considered (Fig. 3). In this figure, the critical values of the internal Grashof number GI_c, are presented as a function of τ for three values of the Prandtl number, $Pr = 5$, 50 and 500 when $GE = 0$ and $r_2 = 0$. The comparison of the neutral curves shows strong thermal effects on the stability such that, as Pr increases, GI_c is monotonically decreased. Apparently, the slope of the neutral curve for $Pr = 5$ is decreasing at $\tau = 7$. This

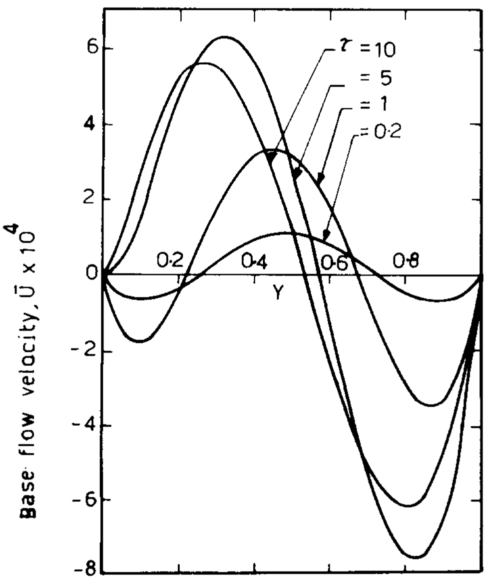


FIG. 2. $\bar{U}$ vs Y for different values of τ ($GE = 0$, $r_2 = 0$).

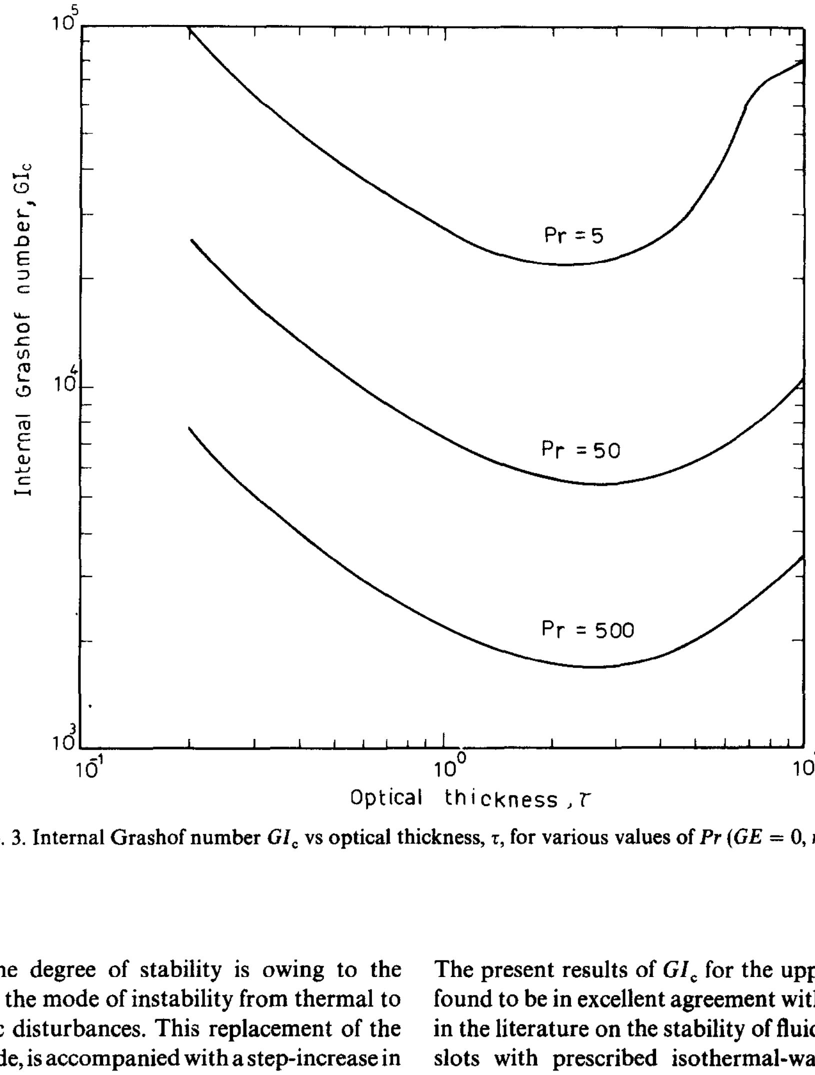


FIG. 3. Internal Grashof number GI_c vs optical thickness, τ, for various values of Pr ($GE = 0$, $r_2 = 0$).

decrease in the degree of stability is owing to the changeover in the mode of instability from thermal to hydrodynamic disturbances. This replacement of the instability mode, is accompanied with a step-increase in the wavenumber a (Fig. 4) and a considerable decay in the wave speed C_i of the critical disturbances as illustrated in Fig. 5.

Figure 6 is prepared to show the variations in the critical internal Grashof number, GI_c vs Pr for the two extremes of a uniform heat source distribution ($Q_v = I_s/h$) and an impulse-function heat source at the transparent boundary ($Q_v = I_s/h$ at $Y = 0$ and $Q_v = 0$ for $Y > 0$). In fact, the first extreme is the lower limit for τ (i.e. $\tau \to 0$) while the latter is the upper limit for τ (i.e. $\tau \to \infty$). From equations (7) and (8) for the base flow functions it can be shown that for $\tau \ll 1$, $GI_c = \overline{GI}_c/\tau$ and for $\tau \gg 1$, $GI_c = \overline{GI}_c * \tau$, where $\overline{GI}_c$ are the critical values at the lower and upper limits, respectively. As shown in Fig. 6 the very optically thin layers are more stable over the whole range of Pr. Furthermore, for very large values of Pr the following asymptotic relations for the two limits of τ are valid:

$$GI_c = \frac{30140}{\tau Pr^{1/2}}\left(\frac{1-r_1 r_2}{1+r_2}\right) \quad \text{for} \quad \tau \to 0 \tag{19}$$

$$GI_c = \frac{9400\tau}{Pr^{1/2}} \quad \text{for} \quad \tau \to \infty. \tag{20}$$

The present results of GI_c for the upper limit of τ are found to be in excellent agreement with those reported in the literature on the stability of fluids inside vertical slots with prescribed isothermal-wall temperatures [16]. However, the comparison of GI_c, for uniform heat sources, with that published by Gershuni *et al.* [14] shows a maximum deviation of about 3.5% as $Pr \to \infty$.

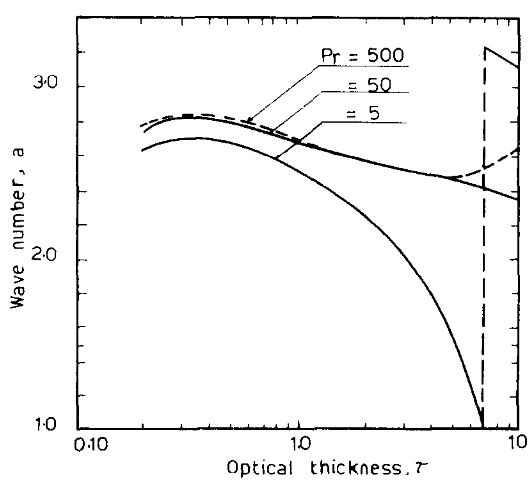


FIG. 4. Wavenumber vs optical thickness, τ, for different values of Pr ($GE = 0$, $r_2 = 0$).

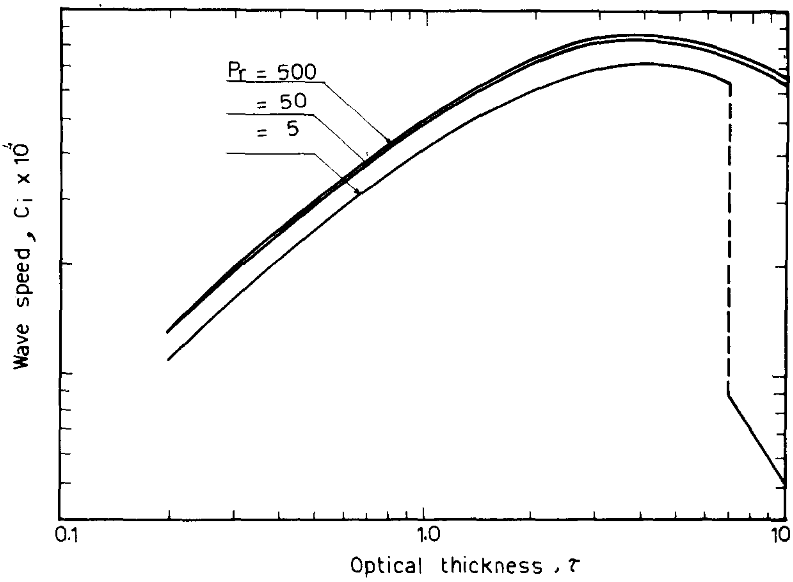


FIG. 5. Wave speed vs optical thickness, τ, for different values of Pr ($GE = 0, r_2 = 0$).

Figure 7 shows the variations in the wavenumber a, and the wave speed C_i of the critical disturbances with the Prandtl number Pr for the case of $\tau \to 0$ with $GE = 0$. The curve for the wavenumber a attains a minimum value of about 2.67 at $Pr = 2.5$. As Pr increases, C_i is monotonically increasing as a result of the destabilizing effect of Pr on the system. However, as $\tau \to \infty$, it is well known that, for $Pr < 12.7$, the instability sets in as stationary horizontal cells with GI_c nearly independent of Pr. On the other hand, for $Pr > 12.7$, the transition occurs in the form of travelling waves with GI_c decreases with Pr as shown in Fig. 6.

Now the effects of the boundary temperature difference expressed by the external Grashof number GE, on the internal Grashof number GI_c are considered for various selected values of τ at $Pr = 5$, as shown in Fig. 8. This external Grashof number that is defined as $GE = \gamma g(T_2 - T_1)d^3/\nu^2$, can be either a positive value or a negative one depending on whether $T_2 > T_1$ or $T_2 < T_1$, where T_1 and T_2 are the temperatures of the transparent and opaque surfaces, respectively. In referring to the curves for $GE < 0$, we conclude that stable equilibrium no longer exists for absolute values of GE higher than 8000. At a fixed value of τ, the degree of stability is first observed to increase and then starts to decline very sharply with increasing GE. As τ increases, the position of the local maxima on the neutral curves is shifted towards the negative direction of GE. This

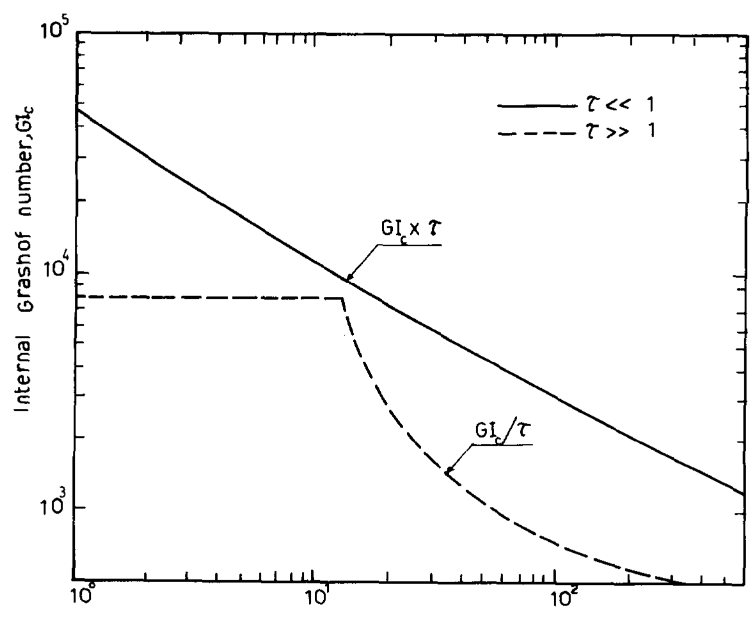


FIG. 6. GI_c vs Pr for very optically thin ($\tau \ll 0$) and very optically thick ($\tau \gg 1$) layers ($GE = 0, r_2 = 0$).

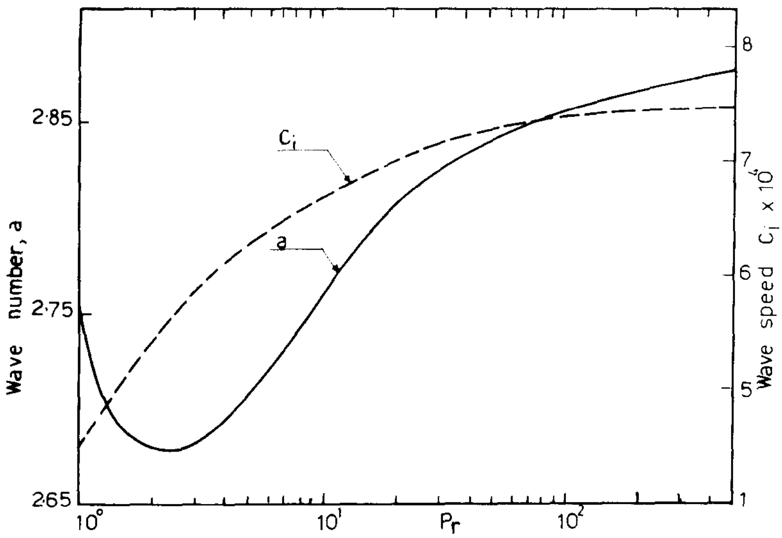


FIG. 7. a and C_i vs Pr for very optically thin layers.

implies that, for optically thinner layers, GE has a stabilizing effect while for optically thicker layers it destabilizes the fluid. On the other hand, when the instability of the fluid is initiated by the effect of negative boundary temperature difference, $GE < 0$, the internal Grashof number GI is found to have a destabilizing effect for all values of τ.

In the case for $GE > 0$, stable regions may exist for values of GE higher than 8000. As is shown, the position of the local maxima is moved towards the positive

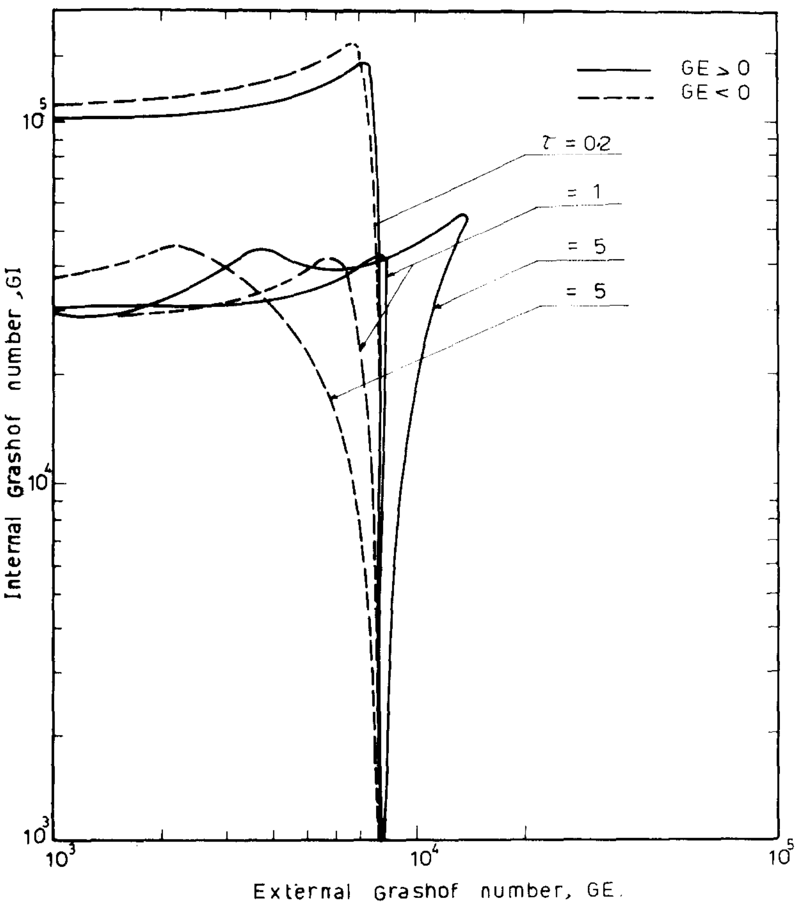
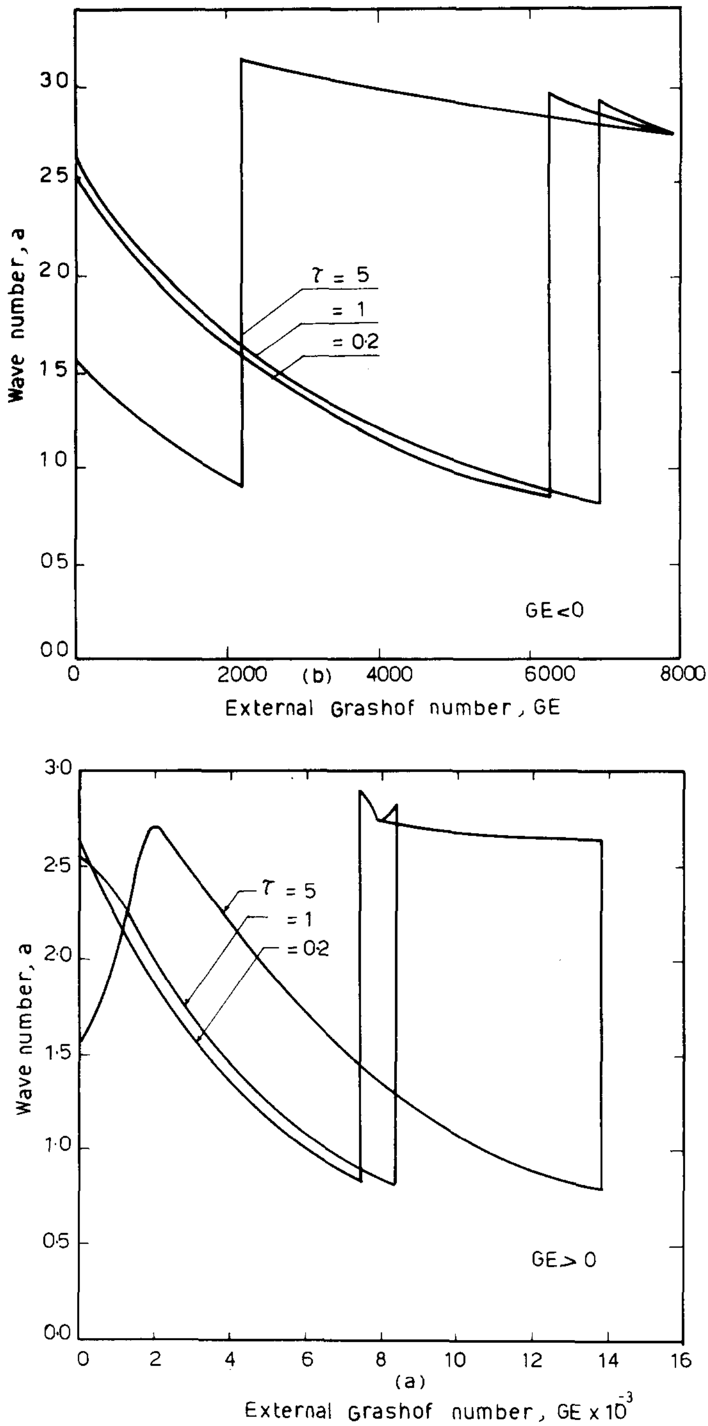


FIG. 8. GI vs GE for the cases of $GE > 0$, $GE < 0$ and for different values of τ ($Pr = 5, r_2 = 0$).



(a)

FIG. 9. a vs GE for different values of τ, $Pr = 5$ for both: (a) $GE > 0$, (b) $GE < 0$.

direction of GE and not the negative direction as in the case for $GE < 0$.

Obviously, the neutral curves for $\tau = 0.2$ and 1 indicate that GE stabilizes the fluid. But in the curve for $\tau = 5$, GE is found to have a stabilizing–destabilizing effect resulting from the transition in the base flow structure from a single cell to two parallel cells, and back as a single cell as GE increases from 0 to 14,000. Eventually, the comparison of the results for the two possible cases of GE corresponding to any values of τ shows that for lower values of GE the degree of stability is higher when $T_2 < T_1$; but for higher values of GE the reverse is true.

The variations of the wavenumber a, and the wave speed C_i at the neutral stability with the external Grashof number GE are plotted in Figs 9 and 10. These

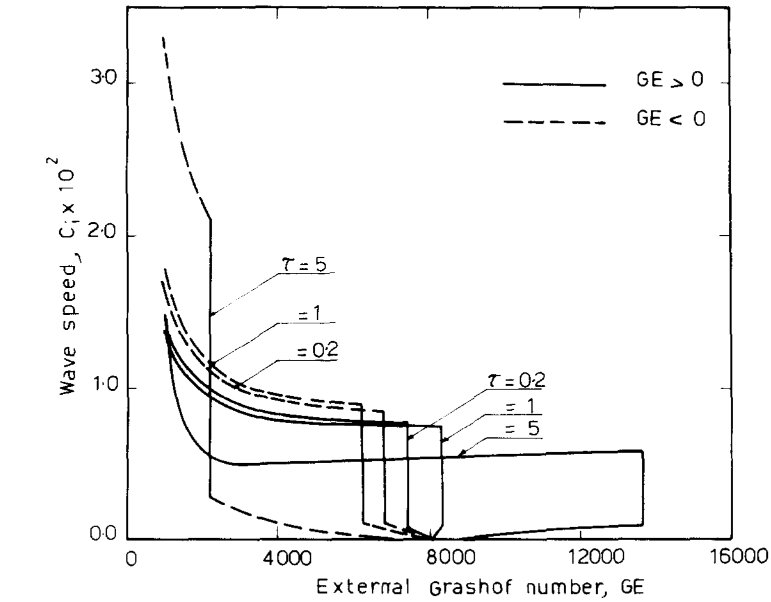


Fig. 10. C_i vs GE for different values of τ, $Pr = 5$ for both: $GE > 0$ and $GE < 0$.

figures are characterized by a jump in the wavenumber a and an associated drop in the wave speed C_i for all selected values of τ. With an increase in τ, the position of the step decrease in both the wavelength $l(2\pi/a)$ and the wave speed C_i is shifted towards the positive direction of GE when $GE > 0$, and vice versa if $GE < 0$. Furthermore, the transition in the wave structures from thermal to hydrodynamic modes is noted to take place at higher values of GE when $GE > 0$.

Figure 11 illustrates the effect of the reflectivities of the boundaries on the stability of the fluid for $GE = 0$ and $Pr = 5$. For small values of τ, the most stable situation is when $r_2 = 0$. In this case, all the radiation reaching the opaque surface ($y = h$) is absorbed and none is reflected. This minimizes the rate of heat generation, resulting in a higher degree of stability. The case of mirror boundaries ($r_1 = r_2 = 1$) corresponds to the most unstable situation, since the radiation is totally absorbed within the fluid. The variations in GI_c with τ for all other combinations of r_1 and r_2 lie between these two limiting curves.

In the limit as $\tau \to 0$, the relation between GI_c and the reflectivities r_1 and r_2 is given by equation (19) and becomes valid for values of τ of order 0.1.

For optically thick layers, the surface reflectivities do not have a significant effect on the stability, since the major part of the radiation is absorbed in the region nearer to the transparent surface and a very small fraction of it reaches the other surface.

Finally, in comparing the results of the present work for liquids with non-uniform volumetric energy sources, with that for radiating gases [19], it is found that the behaviour of instability under the effect of each radiation parameter is opposite to each other. The reason for this is that, for a layer of a radiating gas, radiation tends to drain more energy, by emission, from

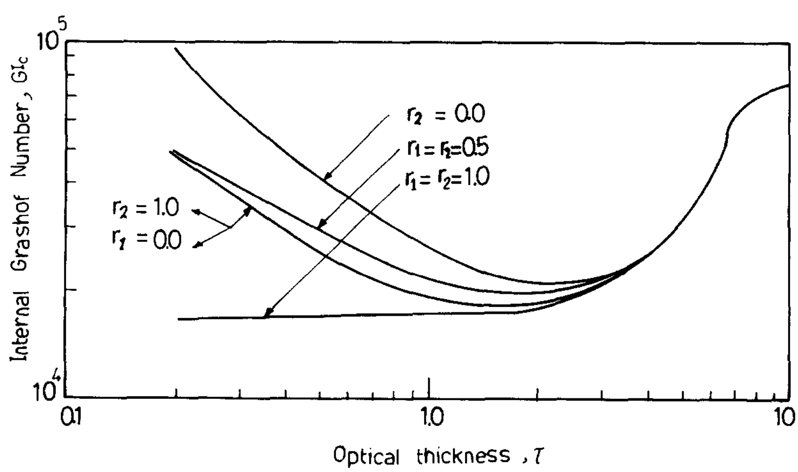


Fig. 11. GI vs τ for various values of reflectivities of the bounding surfaces ($GE = 0$ and $Pr = 5$).

the hotter region to the colder one. This makes the temperature distribution smoother compared to that with no radiation, and as a result the onset of convection is delayed. Conversely, for a vertical layer of liquid with an external radiation source penetrating in its body, radiation tends to enhance the convective motion because of the strong bending of the temperature profiles resulting from the fraction of incident radiation absorbed in the fluid layer.

CONCLUSIONS

The stability results presented in this study in terms of the two independent Grashof numbers GI and GE as functions of the parameters Pr, τ, r_1 and r_2 can be summarized as follows:

(1) As the Prandtl number Pr increases the internal Grashof number GI_c is greatly decreased such that as $Pr \to \infty$, the asymptotic law $GI_c \sim \tau^n \cdot Pr^{-1/2}$ holds. The exponent $n = -1$ and 1 for the extremum cases of very optically thinner and thicker layers respectively under the condition of equal surface temperatures, when ($r_2 = 0$).

(2) The optical thickness of the layer τ has a destabilizing–stabilizing effect on the basic fluid motions as increases over its range studied here. Optically thinner layers are more stable.

(3) The degree of stability defined by GI_c is increased as the external Grashof number GE increases. When $GE > 0$ and with $\tau \gg 1$, the neutral curves attain three turning points in the existed range of GE resulting from the change in the flow structure from single cell, to two parallel cells, and back to a single cell.

(4) Under the conditions where the instability is initiated by GE_c, with an increase in either GI or τ the degree of stability is increased when $T_2 > T_1$ and monotonically decreasing if $T_2 < T_1$. However, the effect of GI on GE_c is less significant than the effect of GE on GI_c.

(5) The surface reflectivities have a significant destabilizing effect only for optically thin layers (i.e. $\tau < 1$).

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LA STABILITE DE LA CONVECTION STATIONNAIRE DANS UNE FENTE VERTICALE ETROITE AVEC DES SOURCES D'ENERGIE NON UNIFORMES ET DES TEMPERATURES DE PAROI INEGALES

Résumé—On résout le problème de la stabilité d'une couche liquide entre deux plaques verticales à températures différentes et soumise à une source externe de rayonnement qui pénètre dans la couche. Les résultats des effets du rayonnement et de la différence de température sont présentés graphiquement par deux nombres de Grashof critiques, différentes fonctions du nombres de Prandtl, de l'épaisseur optique du fluide et des réflectivités des parois. Une justification physique du comportement de l'instabilité est donnée pour un large domaine de variation des paramètres. Les effets des paramètres de rayonnement sur la stabilité des liquides sont trouvés contraires à ceux des gaz dans le même cas isotherme.

DIE STABILITÄT EINER STATIONÄREN KONVEKTIONSBEWEGUNG IN EINEM ENGEN SENKRECHTEN SPALT MIT UNGLEICHMÄSSIG VERTEILTEN INNEREN ENERGIEQUELLEN UND OBERFLÄCHENTEMPERATUREN

Zusammenfassung—Die Stabilität einer Flüssigkeitsschicht wird untersucht, die zwischen zwei auf unterschiedlicher Temperatur gehaltenen, senkrechten Platten eingeschlossen ist und von außen durch Strahlung geheizt wird. Der Einfluß von Strahlung und Randtemperaturdifferenz wird in Form zweier verschiedener kritischer Grashof-Zahlen als Funktion der Prandtl-Zahl, der optischen Fluiddicke und des Reflektionsvermögens der Wände grafisch dargestellt. Für einen weiten Parameter-Bereich wird eine physikalische Deutung des Instabilitäts-Verhaltens angegeben. Zum Schluß wird herausgefunden, daß die Einflüsse der Strahlungsparameter auf die Stabilität von Flüssigkeiten im Gegensatz dazu stehen, was von Gasen für denselben isothermen Fall bekannt ist.

УСТОЙЧИВОСТЬ СТАЦИОНАРНОГО КОНВЕКТИВНОГО ТЕЧЕНИЯ В ВЕРТИКАЛЬНОМ ЩЕЛЕВОМ КАНАЛЕ С НЕРАВНОМЕРНО РАЗМЕЩЕННЫМИ В ОБЪЕМЕ ИСТОЧНИКАМИ ЭНЕРГИИ И РАЗЛИЧНОЙ ТЕМПЕРАТУРОЙ ПОВЕРХНОСТЕЙ

Аннотация—Дано решение задачи об устойчивости слоя жидкости, заключенного между двумя вертикальными пластинами с различнозаданной температурой и подвергаемого воздействию внешнего источника излучения, проникающего внутрь слоя. Результаты по влиянию излучения и разности температур на границах представлены графически в виде зависимости двух различных критических значений числа Грасгофа от числа Прандтля, оптической толщины жидкости и коэффициентов отражения излучения стенкой. Дано физическое обоснование состояния неустойчивости в широком диапазоне значений каждого параметра. Показано, что полученные результаты по влиянию параметров излучения на устойчивость жидкостей не согласуются с данными влияния излучения на устойчивость газов в тех же изотермических условиях.